

⊕ Understand which terms you can add together and which you cannot

$3 + 3$  is two lots of 3 so  $3 + 3 = 2 \times 3$

$a + a$  is two lots of  $a$  so  $a + a = 2 \times a$

We write  $2 \times a$  as  $2a$ .

Just as  $2 \times 3$  and  $4 \times 3$  can be added together to give  $6 \times 3$ ,  $2a$  and  $4a$  can be added together to give  $6a$ . These terms are called **like terms** because they contain the same letter. Like terms can be added together – this is called **collecting** like terms.

Instead of writing  $2m + 3m + b + 2b$ , we can write  $5m + 3b$  or  $3b + 5m$ .

We cannot combine  $5m$  and  $3b$  in any simpler way than this because  $m$  and  $b$  are not like terms – they stand in for different **unknown** numbers.

We can multiply two or more unknowns together.

$a \times b$  is written as  $ab$ .  $b \times a$  is the same as  $a \times b$  and is also written as  $ab$ .

Instead of writing  $2a + 3b + 3ab + ba$ , we can write  $2a + 3b + 4ab$ . We can add  $3ab$  to  $ba$  to make  $4ab$  because  $ab$  and  $ba$  are like terms.

**Example** Simplify the following by collecting like terms:

- a)  $3m + 4m + m + n + 2n$   
b)  $4mv + 2z - 3mv + z + 3m$

a)  $3m + 4m + m + n + 2n = 8m + 3n$

b)  $4mv + 2z - 3mv + z + 3m = mv + 3z + 3m$

There are eight lots of  $m$  and three lots of  $n$ .

First look at the  $mv$  terms:

$$4mv - 3mv = 1mv \text{ (which we usually write as } mv \text{)}$$

Now look at the  $z$  terms:

$$2z + z = 3z$$

Now look at the  $m$  terms: you have  $3m$

## Exercise 6.1

1 Simplify the following expressions by collecting like terms:

- a)  $y + y + y + y + y$   
b)  $m + m + m + b + m$   
c)  $z + a + a + a$   
d)  $p + r + p + p + p + r + r + p + r + p + p + r + p + p + r + p$   
e)  $t + t - q + t - q - q$

2 Simplify the following expressions by collecting like terms:

- a)  $9p + 3r - 2p + r + 2r - p$       b)  $5x + 2y + y - x - y + 7x + 3y$   
c)  $b - t + 3t - t$       d)  $y + 2u + u + 3u + 2u + u - 4y + u - y$   
e)  $x + y - x - y$

3 Simplify the following algebraic expressions:

- a)  $ab + ab + ab + c + c + ab$       b)  $ty - r - r - ty + ty + ty$   
c)  $wd + gh + wd + wd + d - gh - gh$       d)  $abc + ba + ba + ac - abc + bc + abc$   
e)  $xz - yz + xz - yz + z$

Look carefully at each term.

4 Simplify the following algebraic expressions:

- a)  $3r + 2b + 5r + 4b$       b)  $8z + 5y + 7y + 3r + 4z$   
c)  $6t + 7y - t + 5y - 3t$       d)  $6 + 2y - 3 + 4y$   
e)  $16 + t - 2s + 12$       f)  $3w - 6v + 4v - 100$   
g)  $7ab + 4b - 3ab$       h)  $rt + zy + 5rt - 6zy$   
i)  $cx - 10y + 7 + 4x + 11cx + 12$       j)  $abc - 3bc + 1ab + 3ca + 5ca - 2ab$   
k)  $xy + 3 + 4yz - 2zx - 2z + 12xy$       l)  $a + ab + bc + ca + c + b + a + 3bc - 5a - 6c$

5 Copy and complete this addition pyramid. The number in each brick is found by adding the two directly below it. Remember to simplify your answers as much as possible.



6 Write algebraic expressions to represent these situations.

- a) Daniel has  $y$  CDs. Tristan has twice as many. How many do they have altogether?  
b) Alix spends  $£s$  and Heather spends  $£3$  more than Alix. How much do they spend in total?

7 A newsagent uses the following method to calculate the cost of photocopying, in pence:  
number of photocopies  $\times 5$  + number of photocopies  $\times 2$

- a) Replace the phrase 'number of photocopies' with the letter  $a$  and write out the expression.  
b) Simplify this expression.  
c) How would you describe to a sales assistant a simple way of calculating the cost of photocopying?  
d) The newsagent decides to add a set-up charge  $f$  for every batch of copying. Write an algebraic expression for the new cost.  
e) Calculate the cost of making 12 copies of a 4-page brochure, if the set-up charge is 30p for the job.



8 Can you simplify this expression:  $7abc - 5acb + 12bca$ ?  
If so, simplify it.

⊕ Know that algebra follows the same rules as arithmetic

We can multiply two or more unknowns together.

$a \times b$  is written as  $ab$

$a \times b \times c$  is written as  $abc$

$2a \times 3b$  is written as  $6ab$  because  $2 \times a \times 3 \times b = 2 \times 3 \times a \times b$   
 $= 6 \times a \times b$   
 $= 6ab$

$a \times b$  and  $ab$  are

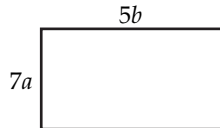
equivalent expressions.

We know that multiplication is commutative, so  $y \times x = x \times y = xy$

We usually write  $y \times x$  as  $xy$ , with the letters in alphabetical order.

We write  $x \div 4$  as  $\frac{x}{4}$ .

**Example** Write an expression using algebra for the area of this rectangle.



Area of a rectangle = base  $\times$  height

Area =  $5b \times 7a$

$= 5 \times 7 \times b \times a$

$= 35ab$

$5a$  and  $7b$  are both **terms**.

$35ab$  is an **expression**.

## Exercise 6.2

1 Write these in a shorter form:

a)  $2a \times 2b$

b)  $3b \times 4a$

c)  $4d \times 8c$

d)  $12m \times 4n$

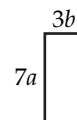
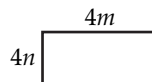
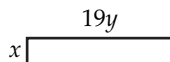
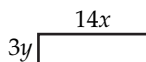
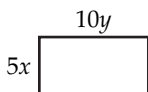
e)  $2h \times i \times g$

f)  $3j \times k \times 4l$

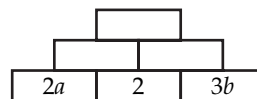
g)  $3y \times 2x \times z$

h)  $3r \times 5t \times 2s$

2 Write expressions using algebra for the area of these rectangles:



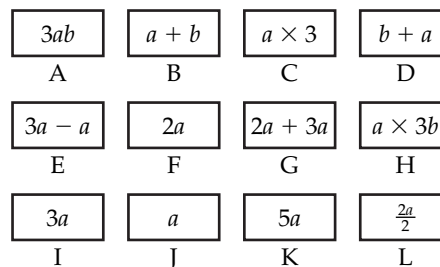
- 3 Copy and complete this multiplication pyramid. The number in each brick is found by multiplying the two directly below it. Remember to simplify your answers as much as possible.



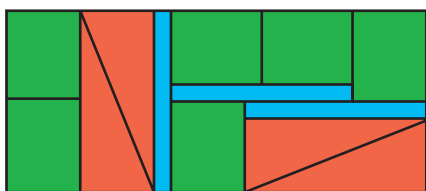
- 4 Copy and complete this multiplication square:

$\times$	2	4	$a$	$b$
$d$				
$2e$				
$3f$				
5				

- 5 a) Write down the pairs of cards that show equivalent expressions.  
 b) Which two cards add together to give  $6a$ ?  
 c) Write a new card that added to K to gives the answer  $10a - 3$ .



- 6 Look at the diagram and the information given below:



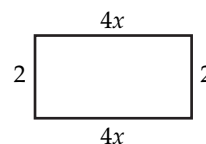
Green rectangles: Base =  $a$ , Height =  $b$   
 Red triangles: Base =  $a$ , Height =  $2b$   
 Blue rectangles: Base =  $2b$ , Height =  $\frac{1}{3}a$

Write an expression for:

- a) the area of a green rectangle      b) the area of a red triangle  
 c) the area of a blue rectangle      d) the total area of the whole shape.  
 e) Write an expression for the area of the whole shape, without adding together the areas of the coloured shapes.

### Investigation

- 7 Amy has a ribbon of length  $8x + 4$ . She wants to use it as a border round some rectangular, triangular and square boxes. Investigate the size and shape of boxes she could use, using whole numbers only. Here is an example:



$$4x + 2 + 4x + 2 = 8x + 4$$

⊕ Substitute values into algebraic expressions

We use letters in **algebra** to represent numbers.

**Substitution** is replacing the letters with numbers.

Look at the expression:  $2A + 3B + 4C$

If  $A = 12$ ,  $B = 10$  and  $C = 7$ , we can calculate the value of the **expression** above by replacing the letters with the numbers. (Remember  $2A$  means  $2 \times A$ .)

$$\begin{aligned} 2A + 3B + 4C &= 2 \times 12 + 3 \times 10 + 4 \times 7 \\ &= 24 + 30 + 28 \\ &= 82 \end{aligned}$$

Remember the order of operations. We multiply before we add.

**Example 1** An algebraic expression is  $5n - 2$ .

- What is the value of the expression if  $n = 10$ ?
- What is the value of the expression if  $n = 112$ ?

$$\text{a) } 5n - 2 = 5 \times 10 - 2$$

$$= 50 - 2$$

$$= 48$$

$$\text{b) } 5n - 2 = 5 \times 112 - 2$$

$$= 560 - 2$$

$$= 558$$

Substitute the value  $n = 10$  into  $5n - 2$ .

**Example 2** Simplify the following algebraic expressions and find their value when  $x = 1$ ,  $y = 5$  and  $z = 10$ .

- $3x + 2y + 3x - 4$
- $4y + 5z - 3z + 3x - xy$

$$\text{a) } 3x + 2y + 3x - 4 = 6x + 2y - 4$$

$$= 6 \times x + 2 \times y - 4$$

$$= 6 \times 1 + 2 \times 5 - 4$$

$$= 6 + 10 - 4$$

$$= 12$$

$$\text{b) } 4y + 5z - 3z + 3x - xy = 4y + 2z + 3x - xy$$

$$= 4 \times y + 2 \times z + 3 \times x - x \times y$$

$$= 4 \times 5 + 2 \times 10 + 3 \times 1 - 1 \times 5$$

$$= 20 + 20 + 3 - 5$$

$$= 38$$

First simplify the expression.

Then substitute in the values.

## Exercise 6.3

- 1 Substitute  $x = 3$ ,  $y = 4$  and  $z = 1$  into the following to find the value of each expression:
- a)  $2x + y$                       b)  $7y - 3x$                       c)  $4z + 2y - 3x$                       d)  $x + y + z$   
 e)  $x - z - 2x$                       f)  $y - z - x$                       g)  $2y + 5z - 10z$                       h)  $3y - 2x - 20z$
- 2 Simplify the following algebraic expressions and find their value when  $x = 10$ ,  $y = 1$  and  $z = 6$ .
- a)  $2y + y$                       b)  $x + y + x + z + y$                       c)  $4x + 2y + 2x$   
 d)  $3x + 2z - z$                       e)  $4x + 5z - 3x + 5z$                       f)  $2x + 4y - x + 7z - 3z - 2y$
- 3 In a garage the cost of servicing a car is calculated like this: Cost in £ =  $25 + p + 15h$   
 $p$  is the cost of parts,  $h$  is the number of hours spent.

Four different cars are brought into the garage on one day. The mechanics note down in a table the cost of parts and the time spent on each car in the table below:

- a) What is the cost of the bill for the Renault owner?  
 b) Which car cost more, the Morris Minor or the Toyota Celica?  
 c) How much change does the BMW owner get from £500?

Car	Cost of parts	Hours
Toyota Celica	£105	3
Renault 5	£56.25	1
Morris Minor	£55.25	4
BMW	£300	3

- 4 Two mobile telephone companies calculate their bills in the following way:

One Chat Bill =  $£15 + 0.25c + 0.1t$   
 All Talk Bill =  $0.5c + 0.05t$

where  $t$  is the number of text messages and  $c$  is total minutes spent on calls.

Work out which company each of these people should choose to get the cheapest bill.

	Name	Minutes spent on calls	Number of text messages
a)	Harriet	45	12
b)	Apu	10	136
c)	Graham	30	50
d)	Jason	120	0



- 5 To find the area of a triangle you use the following formula:

Area =  $\frac{1}{2} \times \text{base} \times \text{height}$  or  $\frac{1}{2}bh$ .

Look carefully at the units.  
 100 cm = 1 m    10 mm = 1 cm

Find the area of the following triangles by substituting the values into the formula:

- a) base = 12 m, height = 5 m                      b) base = 0.5 cm, height = 7 cm  
 c) height = 50 mm, base = 12 cm                      d) height = 3 m, base = 400 cm

### Investigations

- 6 A farmer wishes to enclose a rectangular area of a field for his sheep.

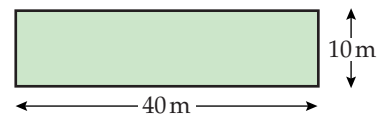
He has 100 m of fencing. He could enclose an area like this:

The perimeter is 100 m.

We can calculate the area using the formulae:

Area = base  $\times$  height.

Area =  $10 \text{ m} \times 40 \text{ m} = 400 \text{ m}^2$ .



- a) Find the base and height of the rectangle that encloses the largest possible area.  
 b) Repeat part a) using different lengths of fencing. What do you notice?

- 7  $r$ ,  $s$  and  $t$  have whole number values. Find values that satisfy the following equation:  $2r + 4s - t = 46$

For example,  $2 \times 10 + 4 \times 8 - 6 = 46$  where  $r = 10$ ,  $s = 8$  and  $t = 6$ .

There are many possible sets of values. Find as many as you can.

Try fixing  $r$  and finding values of  $s$  and  $t$ .

- ⊕ Revise how to simplify expressions
- ⊕ Learn how to multiply out brackets

To **simplify** an algebraic expression you collect together **like terms**.

For example:

$$4a + 5t - a + 4a - 2t + 7t = 7a + 10t$$

There are seven lots of  $a$  and ten lots of  $t$

Sometimes we are given algebraic expressions like  $2(x + 5)$  to simplify.

To simplify expressions like this we can **expand** (remove) the **brackets**.

The number outside the bracket multiplies **every value inside the bracket**.

$$\begin{aligned} \text{For example: } 2(x + 5) &= 2 \times x + 2 \times 5 \\ &= 2x + 10 \end{aligned}$$

**Example 1** Simplify the following by expanding the brackets:

a)  $2(x + 4)$

b)  $3(4 + 2x)$

$$\begin{aligned} \text{a) } 2(x + 4) &= 2 \times x + 2 \times 4 \\ &= 2x + 8 \end{aligned}$$

$$\begin{aligned} \text{b) } 3(4 + 2x) &= 3 \times 4 + 3 \times 2x = 12 + 3 \times 2 \times x \\ &= 12 + 6x \end{aligned}$$

**Example 2** Simplify the following expression and find its value when  $x = 4$  and  $m = 3$ .

$$3x - 2m + 4(x - 3) + 17$$

$$\begin{aligned} 4(x - 3) &= 4 \times x - 4 \times 3 \\ &= 4x - 12 \end{aligned}$$

First expand the bracket.

$$\begin{aligned} \text{So: } 3x - 2m + 4(x - 3) + 17 &= 3x - 2m + 4x - 12 + 17 \\ &= 7x - 2m + 5 \end{aligned}$$

Now collect together like terms.

$$= 7 \times 4 - 2 \times 3 + 5$$

$$= 28 - 6 + 5$$

$$= 27$$

Now substitute the values for  $x$  and  $m$  into the expression.

$$\text{Check: } 3x - 2m + 4(x - 3) + 17 = 3 \times 4 - 2 \times 3 + 4(4 - 3) + 17$$

$$= 12 - 6 + 4 \times 1 + 17$$

$$= 12 - 6 + 4 + 17$$

$$= 27 \checkmark$$

## Exercise 6.4

1 Expand the brackets in the following algebraic expressions:

a)  $3(t + 5)$

b)  $4(x + 6)$

c)  $12(12 + b)$

d)  $3(x - 10)$

e)  $2(x - 24)$

f)  $7(1 - p)$

g)  $3(2x + 4)$

h)  $4(5p - 10)$

i)  $6(3 + 6m)$

j)  $20(1 + 10x)$

k)  $26(1 - t)$

l)  $11(3 - 7b)$

2 Expand the brackets and simplify the following expressions:

- a)  $2(5 + 2c - 4)$                       b)  $4(8x + 3 - 4)$   
 c)  $8(6m + 2q - 5 - q)$             d)  $7(3 + t + 2p - p)$   
 e)  $6(4r - 2t + 10r)$                 f)  $11(12y - 2y + 40)$

You can simplify the expression in the brackets first.

3 In each of the questions below expand the bracket, simplify the expression and find its value by substituting the values given.

- a)  $2 + 6m + 4(m + 3)$                  $m = 4$   
 b)  $8(r - 5) + 12$                          $r = 7$   
 c)  $11 + 2(x - 5) + 10x$                  $x = 12$   
 d)  $10(4 + 2y + z) - 7z$                  $y = 6, z = 3$   
 e)  $8 + 7(j - k) + 12k - 2m$              $j = 8, k = 0, m = 8$



4 The cost of repairing a car is calculated like this:

$$\text{Cost} = 1.02(P + L)$$

$P$  is the cost of parts and  $L$  is the cost of labour.

- a) Expand the brackets and calculate the bill if the parts cost £50 and the labour costs £65.  
 b) Substitute the values for  $P$  and  $L$  into the formula and calculate the cost, without expanding the brackets. Which method is quicker?



5 The amount of tax paid is calculated like this:  $\text{Tax} = 0.25 (\text{Yearly Income} - 4000)$   
 Calculate the amount of tax the following people pay:

- a) a man who earns £35 000 a year  
 b) a woman who earns £1200 a month  
 c) a woman who earns £450 a week and works 47 weeks each year  
 d) a man who earns £4.50 an hour and works 35 hours a week, 48 weeks each year.

6 Each expression in Column A has an equivalent one in Column B. Find the pairs that are equivalent:

Column A	Column B
a) $3(x + y)$	i) $6y + 12$
b) $2(3 - x)$	ii) $6 - 2x$
c) $3(x - 2y)$	iii) $6y + 6$
d) $3(2y + 2)$	iv) $3x + 3y$
e) $2(x - y)$	v) $2x - 2y$
f) $6(y + 2)$	vi) $3x - 6y$

### Investigation

7 The following calculation is set for a maths homework.

'Work out:  $2x + 3(x - 5) - 2y + 10$  where  $x = 5$  and  $y = 2$ .'

Some of the incorrect answers given are below:

*Federica:*

$$\begin{aligned} 2x + 3(x - 5) - 2y + 10 \\ = 2 \times 5 + 3(5 - 5) - 2 \times 2 + 10 \\ = 10 + 0 - 2 \times 2 = 10 - 4 \\ = -14 \end{aligned}$$

*India:*

$$\begin{aligned} 2x + 3(x - 5) - 2y + 10 \\ = 2x + 3x - 5 - 2y + 10 \\ = 2 \times 5 + 3 \times 5 - 5 - 2 \times 2 + 10 \\ = 10 + 15 - 5 - 4 + 10 \\ = 26 \end{aligned}$$

- a) Find the correct answer to the calculation.  
 b) By looking at Federica and India's answers, find out what each of them did wrong and write down some advice you would give them.



⊕ Know the difference between an equation, function, formula and expression.

You already know what an **algebraic expression** looks like:

For example:  $3x + 4t - 7$

An **equation** is a mathematical way of saying two things are equal and includes an equals sign:

For example:  $x + 5 = 12$

We can **solve** this equation to find the value of  $x$ .

A **formula** is a general rule that expresses a relationship between unknowns.

For example: Speed = Distance  $\div$  Time

We could write this as  $S = D \div T$

We can substitute values into this formula. If we know two of the unknowns, we can find the value of the third unknown.

For example: if Distance = 12 metres and Time = 3 seconds

$$\text{Speed} = 12 \div 3 = 4 \text{ metres per second}$$

A **function** is a way of expressing a relationship between two sets of values:

For example:  $x \rightarrow \boxed{\times 3} \rightarrow \boxed{-4} \rightarrow y$

$y$  is always 4 less than triple  $x$ .

We can express this function algebraically as:  $y = 3x - 4$

For any value of  $x$ , we can calculate the value of  $y$ .

**Example 1**  $m = 3n - 5$

- Find the value of  $m$  when  $n = 25$ .
- Find the value of  $m$  when  $n = 10$ .

This is a function. Substitute in the value of  $n$  to find the related value of  $m$ .

a) $m = 3n - 5$	b) $m = 3n - 5$
$= 3 \times 25 - 5$	$= 3 \times 10 - 5$
$= 75 - 5$	$= 25$
$= 70$	

**Example 2** In science we use the formula:  $D = ST$   $D =$  Distance,  $S =$  Speed,  $T =$  Time

- Use the formula to find the distance travelled by a train which travels at a speed of 70 km/h for 2 hours.
- A man walks 20 miles in 2 hours. Calculate his speed.

a) $D = S \times T = 70 \times 2$
$= 140 \text{ km}$
b) $D = S \times T$ So $20 = S \times 2$
$S = 10 \text{ miles per hour}$

We substitute the values:  $D = 20$  and  $T = 2$  into the formula.

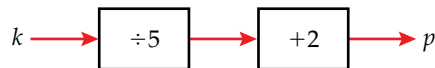
$20 = S \times 2$ :  
 this means 'S multiplied by 2 makes 20'.  
 So S must equal 10.

## Exercise 6.5

- 1 Decide whether the following are algebraic expressions, equations, formulae or functions:
- a)  $2x + 4y$       b)  $x + 7 = 84$       c)  $m = 2n + 5$       d)  $14 = x - 12$
- e)  $2x = y$       f)  $S = \frac{D}{T}$  where  $S = \text{Speed}$ ,  $D = \text{Distance}$  and  $T = \text{Time}$
- g)  $v = m + t$  where  $v = \text{sale price}$ ,  $m = \text{buying price}$  and  $t = \text{profit}$ .



- 2 a) Express this function machine algebraically:  
 b) Express the relationship between  $k$  and  $p$  in words.  
 c) Match the values of  $k$  to the appropriate values of  $p$ .  
 The first one has been done for you.



Values of $k$	0	15	100	87.5	12	150	16
Values of $p$	22	4.4	2	32	5.2	5	19.5

- 3 To convert from US dollars (\$) into British pounds (£) a bank uses the formula:  $D = 1.5P$   
 $D$  is the amount in dollars and  $P$  is the amount in pounds. How many dollars would you get if you converted:
- a) £400      b) £56      c) £1200?      d) Which is worth more – \$1 or £1?

- 4 Copy and complete the following table:

Function	Value of $x$	Value of $y$
$y = 2x - 7$	3	
$y = 6x + 5$	2	
$y = 12 - x$	12	
$y = 4 - 3x$	0	
$y = \frac{3x}{10}$	8	

- 5 Using the formula Distance = Speed  $\times$  Time ( $D = S \times T$ ), find the distance travelled for the following journeys:

- a) A car travels at 40 miles per hour for  $3\frac{1}{2}$  hours.  
 b) A man runs at 10 km per hour for  $1\frac{1}{4}$  hours.  
 c) A snail travels at 3 mm per minute for 20 minutes.  
 d) A cyclist travels 750 m per minute for 50 minutes.

Be careful with the units.



- 6 Using the formula  $D = S \times T$ , find the missing values given the following information:
- a)  $D = 12$  km,  $T = 3$  hours      b)  $D = 550$  m,  $T = 50$  minutes  
 c)  $D = 4$  miles,  $S = 1$  mile per hour      d)  $D = 6$  cm,  $S = 2$  cm per second.

### Investigation

- 7 The area of a rectangle is calculated using the formula:  $A = b \times h$

Where  $A = \text{area}$ ,  $b = \text{base}$ ,  $h = \text{height}$ .

One rectangle with an area of  $100 \text{ m}^2$  is shown:

- a) Draw some other rectangles with the same area.  
 (Use whole numbers for the height and base.)  
 b) Which of your rectangles has the greatest perimeter?  
 c) Can you find a rectangle with a larger perimeter?  
 d) Repeat parts a)–c) for different areas. What do you notice?  
 e) Look back at Q6 in Exercise 6.3. Write a set of instructions for how to get the biggest area or biggest perimeter of a rectangle.

